Electricity and Magnetism

Third Semester BSc Physics

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Contents

Chapter 3

3.1 Introduction

Thousands of years ago, it was noticed that a mineral called magnetite attracted other pieces of magnetite and bits of iron and when small pieces of iron was rubbed with magnetite, the iron began to act like magnetite. When these pieces were free to turn, one end pointed north.

In 1820, Danish physicist Hans Christian Oersted first noticed the magnetic effect of electric current. He found that a pivoted magnetic needle gets deflected when a steady current is passed through a wire kept above or below and parallel to it. Oersted concluded that moving charges or currents produced a magnetic field in the surrounding space. Electrons moving around the nuclei of atoms produce magnetic fields. The motion of these electrons causes some materials, such as iron, to be magnetic. When electric current flows in a wire, electric charges move in the wire. As a result, a wire that contains an electric current also is surrounded by a magnetic field.

Magnetostatics is the study of magnetic fields in systems where the currents are steady (not changing with time). It is the magnetic analogue of electrostatics, where the charges are stationary. Magnetostatics is the study of static magnetic fields. In electrostatics, the charges are stationary, whereas here, the currents are steady or dc(direct current). As it turns out magnetostatics is a good approximation even when the currents are not static as long as the currents do not alternate rapidly.

3.1.1 Magnetic Field and Field Strength B (Magnetic flux density/-Magnetic induction)

Whereas a stationary charge produces only an **electric field E** in the space around it, a moving charges or the current generates, in addition, a **magnetic field B**, again a vector field. It has several basic properties identical to the electric field. It is defined at each point in space (and can in addition depend on time). Experimentally, it is found to obey the principle of superposition: the magnetic field of several sources is the vector addition of magnetic field of each individual source. Magnetic field is a vector field in the neighbourhood of a magnet, electric current, or changing electric field, in which magnetic forces are observable.

3.1.1.1 Lorentz Force

Let us suppose that there is a point charge **q** (moving with a velocity **v** and, located at **r** at a given time **t**) in presence of both the electric field **E(r)** and the magnetic field **B(r)**. The force on an electric charge **q** due to both of them can be written as

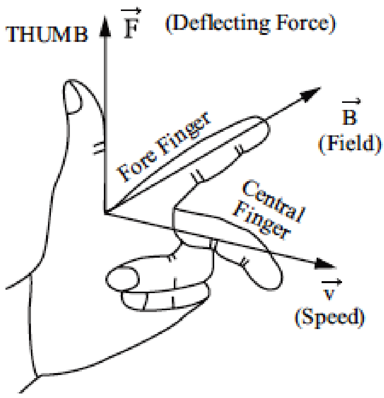
$$F = q\left[E(r) + v \times B(r)\right] \tag{3.1}$$

This force is called the **Lorentz force**. The magnetic force term $F_m = q[v \times B(r)]$ is called the **magnetic Lorentz force** equation.

3.1.1.2 Magnetic induction/Magnetic flux density B

The magnitude of the magnetic field at any point in the field is called the **magnetic field strength, magnetic induction of magnetic flux density, B**. Its SI unit is **weber/square meter** (Wb/m^2) **or tesla (T)**. A smaller unit (non-SI) is **gauss** (= 10^{-4} **tesla)**. The earth's magnetic field is about 3.6×10^{-5} T.

Magnetic induction **B** at a point is said to be one testa if a charge of **1C** moving with a velocity of **1m/s** at right angles to the magnetic induction field **B** at that point, experiences a force of **1N**. The direction of F is perpendicular to the plane containing v and B. It is given by Fleming's left hand rule.



Flemming's left hand rule

3.1.1.3 Magnetic flux ϕ

Magnetic flux is defined as the number of magnetic field lines passing through a given closed surface. It provides the measurement of the total magnetic field that passes through a given surface area. The magnetic flux ϕ through a surface **S** is defined as

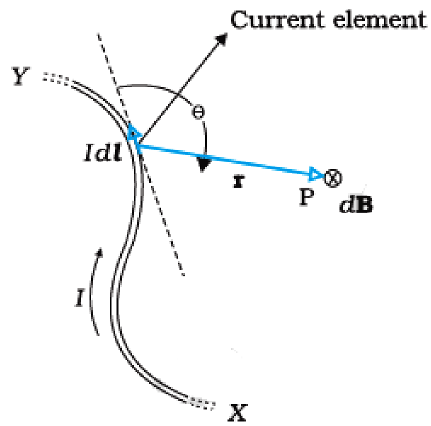
$$\phi = \int_S B \cdot dS \tag{3.2}$$

It represents the lines of induction crossing surface **S**. Magnetic flux is the product of the average magnetic field times the perpendicular area that it penetrates. The SI unit of magnetic flux is a weber (Wb) and CGS unit is maxwell (1 weber = 10^8 maxwell).

3.2 Biot-Savart's law

All magnetic fields that we know are due to currents (or moving charges) and due to intrinsic magnetic moments of particles. Biot-Savart's law gives the relation between current and the magnetic field it produces.

Consider a finite conductor **XY** carrying current **I**. Consider an infinitesimal element **dl** of the conductor. The magnetic field **dB** due to this element is to be determined at a point **P** which is at a distance **r** from it. Let θ be the angle between **dl** and the displacement vector **r**.



Biot-Savart's law

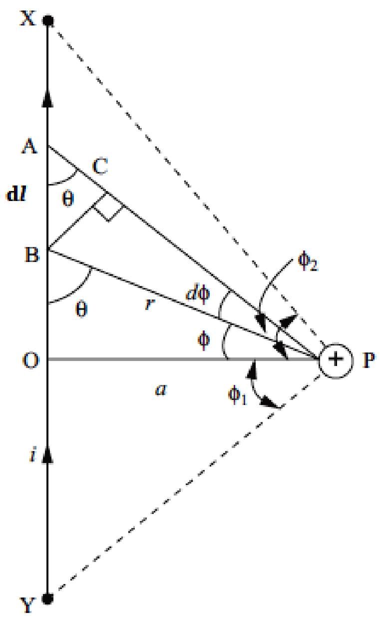
According to Biot-Savart's law, the magnitude of the magnetic field **dB** is proportional to the current **I**, the element length $|dl|$, and inversely proportional to the square of the distance **r**. Its direction is perpendicular to the plane containing **dl** and **r**. Mathematically

$$\begin{aligned} d\vec{B} &= \frac{\mu_0}{4\pi} I \left(\frac{dl \times \hat{r}}{r^2} \right) \qquad (3.3) \\ &= \frac{\mu_0}{4\pi} I \left(\frac{dl \times \vec{r}}{r^3} \right) \\ dB &= \frac{\mu_0}{4\pi} \left(\frac{I dl \sin\theta}{r^2} \right) \end{aligned}$$

where $\mu_0 = 4\pi \times 10^{-7}$ **Tm/A** is called the the **permeability of free space** (or vacuum).

3.2.1 Magnetic field at a point due to a straight conductor

Consider a straight conductor **XY** carrying a current **i** in the direction Y to X. **P** is a point at a perpendicular distance a from the conductor. Consider an element **AB** of length **dl** of the conductor. Let **BP = r** and $\angle BOP = \theta$.



Magnetic induction at P, due to the current element AB is

$$dB = \frac{\mu_0}{4\pi}\left(\frac{Idl\sin\theta}{r^2}\right)$$

From the figure, $\angle OPB = \phi$, $\angle BPA = d\phi$ and $BC = dl\sin\theta = rd\phi$. From $\triangle OPB$, $\cos\phi = a/r$ and $r = a/\cos\phi$. So

$$dB = \frac{\mu_0}{4\pi}\left(\frac{I\cos\phi d\phi}{a}\right)$$

The direction of **dB** at **P** will be given by the right hand rule.
Let ϕ_1 and ϕ_2 be the angles made by the ends of the wire at P. Then, magnetic induction at P due to the whole conductor is given by

$$B = \int_{\phi_1}^{\phi_2}\frac{\mu_0}{4\pi}\left(\frac{I\cos\phi d\phi}{a}\right)$$

or

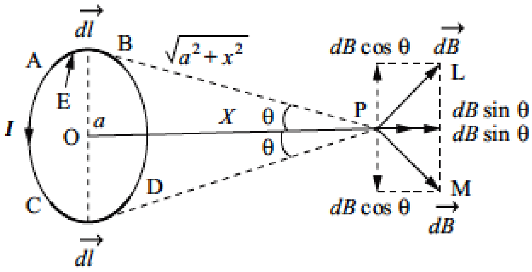
$$B = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\sin\phi_1 + \sin\phi_2\right) \tag{3.4}$$

If the conductor is infinitely long, $\phi_1 = \phi_2 = 90^0$. Then

$$B = \frac{\mu_0}{4\pi}\frac{2I}{a}$$

3.2.2 Magnetic field at an axial point due to a current carrying circular coil

Consider a circular coil of radius **a**, carrying a current **I**. We assume that the current **I** is steady and that the evaluation is carried out in free space (i.e., vacuum). **P** is a point on its axis at a distance **x** from the centre **O**. Consider two opposite current elements **AB** and **CD** each of length **dl**. The distance of P from any point on the circumference of the coil is $\sqrt{a^2 + x^2}$.



The field at P due to AB is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{(a^2+x^2)}$$

This is in the direction PL, perpendicular to the line joining the midpoint of AB with P. Considering the element CD, the magnitude of dB at P due to this element is the same as that given by the above equation, but, it is directed along PM.

We have from the figure $\angle EPO = \theta$ and $\sin\theta = \frac{a}{\sqrt{(a^2+x^2)}}$. The components $\boldsymbol{dB\cos\theta}$ perpendicular to the axis of the coil and due to the two opposite elements cancel each other. But components $\boldsymbol{dB\sin\theta}$ along the axis are in the same direction. Thus, the total magnetic induction at P due to the entire coil is

$$\begin{aligned} B &= \int_{l=0}^{l=2\pi a} dB\sin\theta \qquad (3.5)\\ &= \int_{l=0}^{l=2\pi a} \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{(a^2+x^2)} \\ &= \frac{\mu_0}{4\pi}\frac{2\pi I a^2}{(a^2+x^2)^{3/2}} \end{aligned}$$

If the coil has N turns, then

$$B = \frac{\mu_0}{4\pi}\frac{2\pi N I a^2}{(a^2+x^2)^{3/2}} \qquad (3.6)$$

When P is at a large distance from the center of the coil, $\boldsymbol{x >> a}$

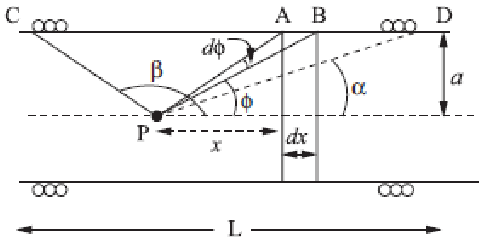
$$B = \frac{\mu_0}{4\pi}\frac{2\pi N I a^2}{x^3} \qquad (3.7)$$

At the center of the coil **x=0**. So

$$B = \frac{\mu_0 N I}{2a} \qquad (3.8)$$

3.2.3 Magnetic field strength at the axis of a solenoid carrying current

Let **L** be the length of the solenoid and **N** the total number of turns in its winding. The number of turns per unit length is then **N/L**. **a** is the radius of the solenoid. A current **I** is flowing in the solenoid. The solenoid contains *air* in its *core*. Let **P** be an axial point of the solenoid at which the magnetic induction **B** is to found out.



Consider an elementary length **dx** of the solenoid, at a distance **x** from **P**. This element **AB** can be regarded as a circular coil of radius **a** containing **N dx/L** turns.

Magnetic induction at **P** due to the element **dx** is

$$dB = \frac{\mu_0 I}{2} \frac{N dx}{L} \frac{a^2}{(a^2 + x^2)^{3/2}} \tag{3.9}$$

From the figure, $x = a \cot \phi$ and $dx = -a\ cosec^2\phi\ d\phi$. substituting these values in equation 3.9 and simplifying, we get

$$dB = -\frac{\mu_0 IN}{2L} \sin \phi\ d\phi \tag{3.10}$$

The magnetic induction at P due to the entire length of the solenoid will be obtained by the above expression from α to β.

$$\begin{aligned} B &= -\frac{\mu_0 IN}{2L} \int_\alpha^\beta \sin \phi\ d\phi \qquad (3.11) \\ &= \frac{\mu_0 IN}{2L} (\cos \alpha - \cos \beta) \end{aligned}$$

B is directed parallel to the axis of the solenoid.
At a point well inside a very long solenoid, $\alpha = 0^0, \beta = 180^0$

$$B = \frac{\mu_0\ NI}{L} \tag{3.12}$$

At an axial point at one end of a long solenoid $\alpha = 0^0, \beta = 90^0$

$$B = \frac{\mu_0\ NI}{2L} \tag{3.13}$$

If the core of solenoid is not of air, but of some material of permeability $\boldsymbol{\mu = \mu_0 \mu_r}$, then

$$B = \frac{\mu IN}{2L} (\cos \alpha - \cos \beta) = \frac{\mu_0 \mu_r IN}{2L} (\cos \alpha - \cos \beta) \tag{3.14}$$

3.3 Divergence and curl of magnetic field

The static magnetic field **B(x, y, z)** — such as the field of steady currents — obeys the equations

$$\nabla \cdot B = 0 \tag{3.15}$$
$$\nabla \times B = \mu_0 J \tag{3.16}$$
$$(3.17)$$

The zero-divergence equation $\nabla \cdot B = 0$ is valid for any magnetic field, even if it is time-dependent rather than static. Physically, it means that there are no magnetic charges. Consequently, the magnetic field lines never begin or end anywhere in space; instead they form closed loops or run from infinity to infinity. Thus, a magnetic field has no divergence which is a mathematical statement that there are no magnetic monopoles. The curl of a magnetic field around an axis is proportional to the component of the current density along the axis.

3.3.1 Divergence of B or Gauss' Law for Magnetic Fields

Gauss's law for magnetism states that no magnetic monopoles exists and that the total flux through a closed surface must be zero.

From Biot-Savart's law, the magnetic field produced by a current element $I\vec{dl}(x', y', z')$ at a point $P(x, y, z)$ whose distance from the current element $\vec{r} = (x - x')\hat{i} + (y - y')\hat{j} + (z - z')\hat{k}$, is given by

$$dB = \frac{\mu_0}{4\pi}\frac{I\vec{dl} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\vec{dl} \times \vec{r}}{r^3} \tag{3.18}$$

Therefore, the magnetic field at P due to the whole current loop is given by

$$\oint d\vec{B} = \oint \frac{\mu_0}{4\pi}\frac{I\vec{dl} \times \vec{r}}{r^3} \tag{3.19}$$

Taking divergence both sides, we get

$$\nabla \cdot \vec{B} = \frac{\mu_0 I}{4\pi}\oint \nabla \cdot \left(\frac{\vec{dl} \times \vec{r}}{r^3}\right) \tag{3.20}$$

We have

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

Therefore

$$\nabla \cdot \vec{B} = \frac{\mu_0 I}{4\pi}\oint (\frac{\vec{r}}{r^3}) \cdot (\nabla \times \vec{dl}) - \vec{dl} \cdot (\nabla \times \frac{\vec{r}}{r^3}) \tag{3.21}$$

$\boldsymbol{\vec{B}}$ is a function of (x, y, z) and $\boldsymbol{I\vec{dl}}$ is a function of (x', y', x'). The integration is over the primed coordinates (x', y', z'). The divergence is to be taken with respect to the unprimed coordinates (x, y, z).

Since $\vec{dl}$ is not a function of (x, y, z), we have, $\nabla \times \vec{dl} = 0$.
Also

$$\nabla \times \frac{\vec{r}}{r^3} = -\nabla \times \frac{1}{\vec{r}}$$

and $\nabla \times \nabla = 0$. So

$$\nabla \times \frac{\vec{r}}{r^3} = 0$$

Using these, we get, the divergence of $\vec{B}$ as

$$\boxed{\nabla \cdot \vec{B} = 0} \tag{3.22}$$

Any vector whose divergence is zero is known as a **solenoidal vector**. Thus, magnetic field vector $\vec{B}$ is a solenoidal vector. Equation 3.22 is also known as the differential form of **Gauss's law** in magnetism.

3.3.1.1 Integral form of Gauss' law in magnetism

By equation 3.22, we have

$$\nabla \cdot \vec{B} = 0$$

Therefore

$$\int_{vol} (\nabla \cdot \vec{B})\, dV = 0 \tag{3.23}$$

Using Gauss divergence theorem, equation 3.23 can be written as

$$\int_{vol} (\nabla \cdot \vec{B})\, dV = \oint_{surface} \vec{B}.\vec{dS} = 0 \tag{3.24}$$

Thus

$$\oint_{surface} \vec{B}.\vec{dS} = 0 \tag{3.25}$$

Equation 3.25 says that the total magnetic flux $\phi_B = \oint_{surface} \vec{B}.\vec{dS}$ over any closed surface is zero, which is the integral form of **Gauss' law in magnetism**.

The equation states that there is no net magnetic flux (which can be thought of as the number of magnetic field lines through an area) that passes through an arbitrary closed surface. This means the number of magnetic field lines that enter and exit through this closed surface is the same. This is explained by the concept of a magnet that has a north and a south pole, where the strength of the north pole is equal to the strength of the south pole. This is equivalent to saying that a magnetic monopole, meaning a solitary north or south pole, does not exist because for every positive magnetic pole, there must be an equal amount of negative magnetic poles.

3.3.2 Curl of magnetic induction $\vec{B}$

From Ampere's circuital theorem, we have

$$\oint_l \vec{B} \cdot \vec{dl} = \mu_0 I \tag{3.26}$$

Consider a region of space in which currents are flowing, the **current density $\vec{J}$** varies from point to point but is time-independent. The total steady current **I** is given by

$$I = \int_{surface} \vec{J} \cdot \vec{dS} \tag{3.27}$$

where $\vec{dS}$ is the elemental area vector of the surface **S** bounded by the closed path.
Substituting equation 3.27 in equation 3.26,

$$\oint_l \vec{B} \cdot \vec{dl} = \mu_0 \int_{surface} \vec{J} \cdot \vec{dS} \tag{3.28}$$

But according to **Stoke's theorem**, the surface integral of the curl of a function over a surface bounded by a closed surface is equal to the line integral of the particular vector function around that surface. Therefore, we can write

$$\oint_l \vec{B} \cdot \vec{dl} = \int_{surface} (\nabla \times \vec{B}) \cdot \vec{dS} \tag{3.29}$$

Comparing equations 3.28 and 3.29, we get

$$\int_{surface} (\nabla \times \vec{B}) \cdot \vec{dS} = \mu_0 \int_{surface} \vec{J} \cdot \vec{dS} \tag{3.30}$$

From equation 3.30, it is clear that

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}} \tag{3.31}$$

Equation 3.31 gives the curl of magnetic field, which is the **differential form of Ampere's circuital theorem**.

3.4 Magnetic vector potential A

From the magnetic form of Gauss's law

$$\nabla \cdot \vec{B} = 0$$

it is evident that the magnetic flux density $\vec{B}$ is a solenoidal vector field.

A solenoidal field can be written as the curl of some other vector field. Since the divergence of any curl is zero, it is reasonable to assume that the magnetic flux density may be written

$$\vec{B} = \nabla \times \vec{A} \tag{3.32}$$

$\vec{\boldsymbol{A}}$ refers to magnetic potential and is called the **vector magnetic potential**. Magnetic induction is given by the curl of vector magnetic potential. The vector magnetic potential $\vec{\boldsymbol{A}}$ is defined as the vector, whose curl at any point gives the value of the magnetic field $\vec{\boldsymbol{B}}$ at that point.
The only other requirement placed on $\vec{\boldsymbol{A}}$ is that

$$\nabla \times \vec{B} = \nabla \times (\nabla \times \vec{A}) = \mu_0 J \tag{3.33}$$

Unit of **A** is Wb/m or T.m

Vector field **A** is called the magnetic vector potential because of its analogous function to the electric scalar potential **V**.

We can add any term whose curl is zero to the vector potential and it still gives the same magnetic field. Unlike **V** (electric scalar potential), **A** does not have a physical significance. It serves as a convenient intermediate step for the computation of **B**.

In other words, given some vector field **B**, the solution **A** to the differential equation $B = \nabla \times A$ is not unique. To completely (i.e., uniquely) specify a vector field, we need to specify both its divergence and its curl. For simplicity we can take magnetic vector potential **A** as if its divergence$\nabla \cdot A$ is equal to zero.

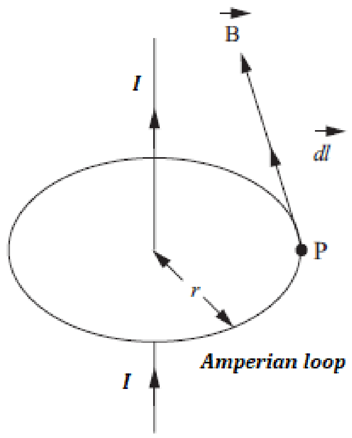
3.5 Ampere's circuital law

Ampere's circuital law relates the integral of the magnetic field around a closed loop to the electric current passing through the loop. Ampere's circuital law can be written as the line integral of the magnetic field surrounding closed-loop equals the μ_0 times the algebraic sum of currents passing through the loop. That is

$$\oint_l \vec{B} \cdot \vec{dl} = \mu_o \, I \tag{3.34}$$

where **I** is the total current through the surface. The integral is taken over the closed loop coinciding with the boundary **C** of the surface. The relation above involves a sign-convention, given by the right-hand rule. Let the fingers of the right-hand be curled in the sense the boundary is traversed in the loop integral $\oint B \cdot dl$. Then the direction of the thumb gives the sense in which the current **I** is regarded as positive.
For several applications, a much simplified version of equation 3.34 proves sufficient. In such cases, it is possible to choose the loop (called an *amperian* loop) such that at each point of the loop, either **B** is tangential to the loop and is a non-zero constant B, or B is normal to the loop, or B vanishes.

Proof



Consider a long straight conductor carrying a current **I** perpendicular to the page directed outward. According to Biot-Savart law, the magnitude of the magnetic induction at a distance **r** from it is given by

$$B = \frac{\mu_0 I}{2\pi r} \tag{3.35}$$

At each point on this circle, $\vec{B}$ has a constant magnitude B and **dl** which is always tangential to the path of integration, points in the same direction as **B** (so that angle between them is zero degree). Thus

$$\oint_l B \cdot dl = B \oint_d l = B \times 2\pi r \tag{3.36}$$

where $\oint_l dl = 2\pi r$.
Substituting equation 3.35 in 3.36, we get

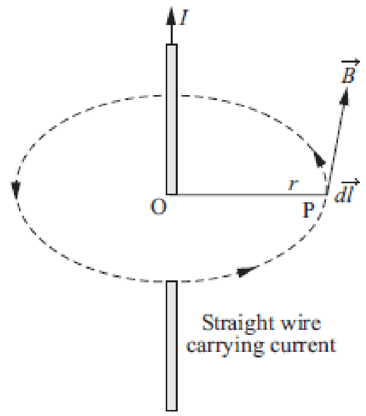
$$\oint_l B \cdot dl = (\frac{\mu_0 I}{2\pi r}) \times 2\pi r = \mu_o I \tag{3.37}$$

Differential Form of Ampere's Law is

$$\boldsymbol{\nabla \times B = \mu_0 J}$$

3.5.1 Magnetic induction due to long straight current carrying wire

Consider a long straight wire carrying a current **I**. The magnetic lines of force are concentric circles centred on the wire. Let **P** be a point distant **r** from the wire, at which the magnetic filed strength **B** is to be found out.



Consider a circular path of radius **r** passing through **P**. This is the amperian loop. By symmetry, the value of magnetic field **B** is same at each point on the circular path. Consider a small element **dl** of a line of amperian loop at P. **B** and **dl** are always directed along the same direction and the angle between **B** and **dl** is 0^0. Line integral of **B** along the boundary of circular path

$$\oint_l B \cdot dl = B \oint_l dl = B2pir$$

From Ampere's circuital law,

$$\oint_l B \cdot dl = \mu_0 \times \text{current enclosed by the path}$$

From the above equations,

$$B2\pi r = \mu_o I$$

Or

$$B = \frac{\mu_0 I}{2\pi r} \tag{3.38}$$

3.6 Magnetic properties of materials

Magnetic phenomena are universal in nature. The word magnet is derived from the name of an island in Greece called magnesia where magnetic ore deposits were found, as early as 600 BC. The directional property of magnets was also known since ancient times. A thin long piece of a magnet, when suspended freely, pointed in the north-south direction.

The earth behaves as a magnet with the magnetic field pointing approximately from the geographic south to the north. When a bar magnet is freely suspended, it points in the north-south direction. The tip which points to the geographic north is called the north pole and the tip which points to the geographic south is called the south pole of the magnet. There is a repulsive force when north poles (or south poles) of two magnets are brought close together. Conversely, there is an attractive force between the north pole of one magnet and the south pole of the other. We cannot isolate the north, or south pole of a magnet. If a bar magnet is broken into two halves, we get two similar bar magnets with somewhat weaker properties. Unlike electric charges, isolated magnetic north and south poles known as magnetic monopoles do not exist. It is possible to make magnets out of iron and its alloys.

Magnetic property refers to the response of a material to an applied magnetic field. The macroscopic magnetic properties of a material are a consequence of interactions between an external magnetic field and the magnetic dipole moments of the constituent atoms. Different materials react to the application of magnetic field differently. The most familiar effects occur in ferromagnetic materials, which are strongly attracted by magnetic fields and can be magnetized to become permanent magnets, producing magnetic fields themselves. Only a few substances are ferromagnetic. The most common ones are iron, cobalt and nickel and their alloys.

3.6.1 Magnetic Induction B

The magnetic induction is defined through its action on a moving charge. If a positive test charge q moving with velocity v through a point in a magnetic field experiences a force F, then the magnetic induction B at that point is defined by

$$F = q(v \times B)$$

The magnetic induction (**B**) in any material is the number of lines of magnetic force passing perpendicular through unit area. Refer section 3.1.1 for details.

3.6.2 Magnetic field intensity H

The magnetic field intensity (**H**) at any point in the magnetic field is the force experienced by a unit north pole placed at that point. Its unit is **A/m**.
The magnetic induction **B** due to a magnetic field of intensity **H** applied in vacuum is

$$B = \mu_0 H \tag{3.39}$$

The permeability of free space has a value of $\mu_0 = 4\pi \times 10^{-7}$ H/m.
If a magnetic field of intensity **H** is applied in a medium, the magnetic induction (**B**) in the medium is given by

$$B = \mu H \tag{3.40}$$

The permeability of the medium $\mu = \mu_r\ \mu_0$. The term

$$\mu_r = \frac{\mu}{\mu_0} \tag{3.41}$$

is called the relative permeability of the medium, which has no units.

3.6.3 Magnetisation M

When a magnetic material is placed in a magnetic field, the elementary current-loops in the material become aligned parallel to the field. The material is then magnetised, and acquires a magnetic dipole moment.

Magnetisation **M** of the material is defined as the magnetic dipole moment induced per unit volume of the material. Unit of **M** is **A/m**. Let **m** be the magnetic dipole moment of a specimen of volume **V**. Then

$$M = \frac{m}{V} \tag{3.42}$$

In an unmagnetized matter **M** will be zero. In a uniformly magnetized matter, each atomic magnetic dipole will point in the same direction and magnetization **M** will be constant throughout.

3.6.4 Relation connecting B, H and M

Consider a material of magnetic permeability μ is placed in a magnetic field B_0. It gets magnetised as per its magnetic properties. Let the field induced inside the material be B. Then

$$B = B_{applied} + B_{induced} \tag{3.43}$$

We have

$$B_{applied} = B_0 = \mu_0 H \tag{3.44}$$

where H is the **magnetising field vector** or **magnetic intensity** and

$$B_{induced} = \mu_0 M \tag{3.45}$$

Thus, the total magnetic field **B** is written as

$$B = \mu_0(H + M) \tag{3.46}$$

So inside the material **B, H** and **M** are related as

$$B = \mu_0(H + M)$$

or

$$H = \frac{B}{\mu_0} - M \tag{3.47}$$

When there is vacuum or no magnetic material, **M=0**, then

$$H = \frac{B}{\mu_0} \tag{3.48}$$

So the total field is due to external factors which is represented by **H** and also due to the specific nature of the magnetic material, namely **M**.

3.6.5 Magnetic permeability μ and susceptibility χ

In electrodynamics, **permeability** is the measure of the resistance of a substance against the formation of a magnetic field in it. The auxiliary magnetic field **H** represents how a magnetic field B influences the organization of magnetic dipoles in a given substance. The magnetic flux density and magnetic intensity are related according to

$$B = \mu H \tag{3.49}$$

Comparing equations 3.46 and 3.49, we get

$$\mu H = \mu_0 (M + H) \tag{3.50}$$

Or

$$\mu = \mu_0 \left(1 + \frac{M}{H}\right) \tag{3.51}$$

Or

$$\mu = \mu_0 \left(1 + \chi_m\right) \tag{3.52}$$

where

$$\chi_m = \frac{M}{H} \tag{3.53}$$

is called the **magnetic susceptibility**. It is a dimensionless proportionality factor that indicates the degree of magnetization of a material in response to an applied magnetic field.

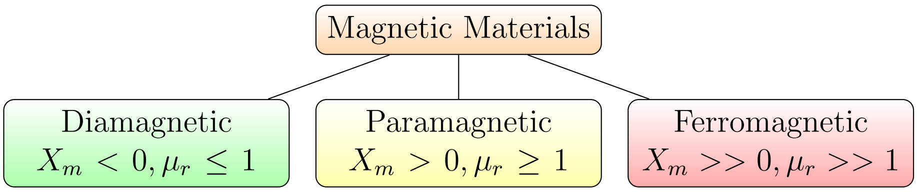
From equations 3.41 and 3.52, we get

$$\mu_r = \frac{\mu}{\mu_0} = (1 + \chi_m) \tag{3.54}$$

μ_r, **relative permeability** (dimensionless) is the ratio of the permeability of the material to the permeability of vacuum.

3.7 Classification of materials based on magnetic properties

Neither μ_r nor χ are constants, as they can vary with the position in the medium. They depend not only on the material but also on the magnitude of the field, **H**, the frequency of the applied magnetic field, humidity, temperature, and other parameters. Nearly all materials respond to a magnetic field by becoming magnetized, but most are **paramagnetic** with a response so faint that it is of no practical use. A few, however, contain atoms that have large dipole moments and have the ability to spontaneously magnetize (i.e. to align their dipoles in parallel). These are called ferromagnetic and ferrimagnetic materials (the second one is called ferrites for short), and it is these that are of real practical use. Ferromagnetic, ferrimagnetic, or antiferromagnetic materials possess permanent magnetization even without external magnetic field and do not have a well defined zero-field susceptibility.
The values of μ_r and χ can be used to classify materials in to paramagnetic, ferromagnetic and diamagnetic.



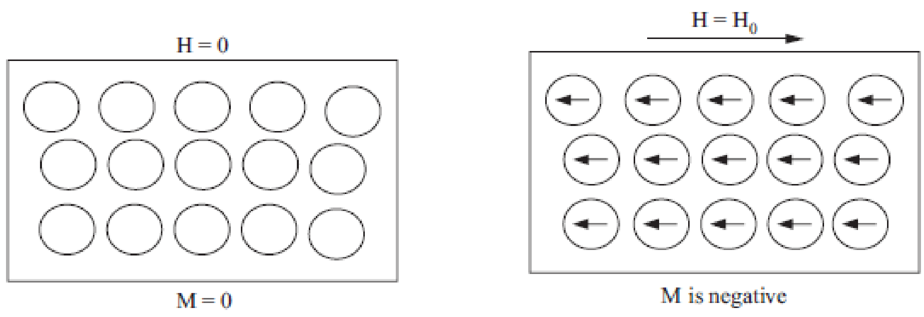
In terms of the susceptibility χ, a material is diamagnetic if χ is negative, para - if χ is positive and small, and ferro - if χ is large and positive.

Diamagnetic	Paramagnetic	Ferromagnetic
Susceptibility is always negative	small positive Susceptibility	large positive Susceptibility
$-1 \leq \chi < 0$	$0 \leq \chi < \epsilon$	$\chi >> 0$
$0 \leq \mu_r < 1$	$1 \leq \mu_r < 1 + \epsilon < 0$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$
repelled	small attraction	large attraction
water, mercury, gold, wood, copper, silicon, organic materials	aluminium, sodium, calcium	iron, cobalt, nickel, gadolinium
universal	-	-
independent of temperature	depends on temperature	depends on temperature
no permanent dipole moment	possesses permanent dipole moment	possesses permanent dipole moment

Here ϵ is a small positive number introduced to quantify paramagnetic materials. A material is said to be **nonmagnetic** if $\chi_m = 0$ (or $\mu_r = 1$); it is magnetic otherwise. *Free space and air are regarded as nonmagnetic.*

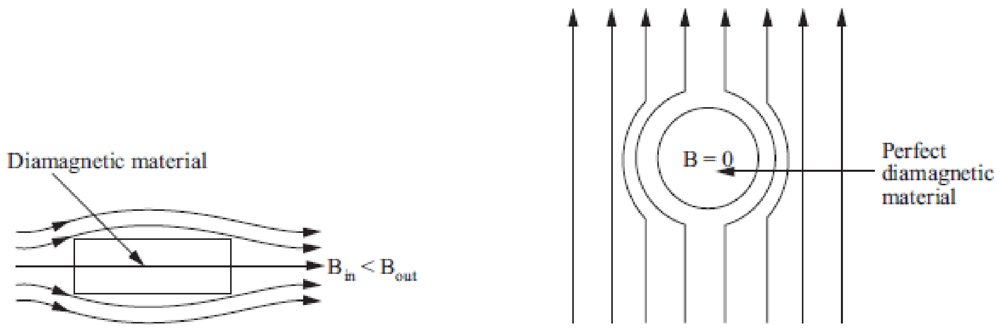
3.7.1 Diamagnetism

Diamagnetic substances are those which have tendency to move from stronger to the weaker part of the external magnetic field. In other words, unlike the way a magnet attracts metals like iron, it would repel a diamagnetic substance. When an external magnetic field is applied, the atoms acquire a small induced magnetic moment in a direction opposite to the direction of applied field. When placed in a non-uniform magnetic field, the bar will tend to move from high to low field. The simplest explanation for diamagnetism is as follows. Electrons in an atom orbiting around nucleus possess orbital angular momentum. These orbiting electrons are equivalent to current-carrying loop and thus possess orbital magnetic moment. Diamagnetic substances are the ones in which resultant magnetic moment in an atom is zero. When magnetic field is applied, those electrons having orbital magnetic moment in the same direction slow down and those in the opposite direction speed up. This happens due to induced current in accordance with Lenz's law. Thus, the substance develops a net magnetic moment in direction opposite to that of the applied field and hence repulsion.



Diamagnetism

Diamagnetism is present in all the substances. However, the effect is so weak in most cases that it gets shifted by other effects like paramagnetism, ferromagnetism, etc. The most exotic diamagnetic materials are superconductors. These are metals, cooled to very low temperatures which exhibits both perfect conductivity and perfect diamagnetism. Here the field lines are completely expelled. $\chi = -1$ and $\mu_r = 0$. A superconductor repels a magnet and is repelled by the magnet. The phenomenon of perfect diamagnetism in superconductors is called the **Meissner effect**. Superconducting magnets can be gainfully exploited in variety of situations, for example, for running magnetically levitated superfast trains.



Behaviour of magnetic field lines near a diamagnetic substance

3.7.2 Paramagnetism

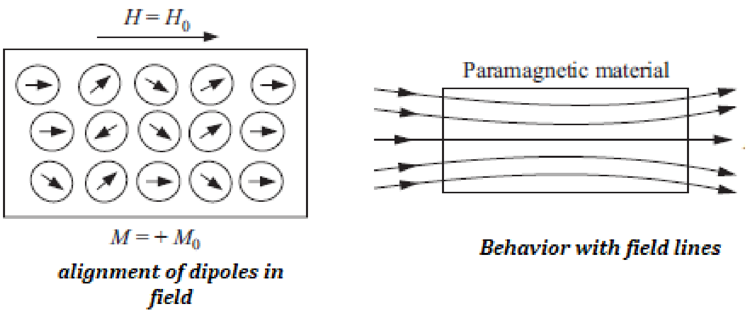
Paramagnetic materials possess permanent magnetic dipoles. In the absence of an external applied field, the dipoles are randomly oriented. Hence the net magnetization in any given direction is zero. Paramagnetic substances are those which get weakly magnetised when placed in an external magnetic field. They have tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get weakly attracted to a magnet.

In the absence of external magnetic field, the individual atoms of paramagnetic material possess a permanent magnetic dipole moment of their own. Each atom possesses a permanent magnetic moment. When $\mathbf{H} = \mathbf{0}$, all the magnetic moments are randomly oriented because of the ceaseless random thermal motion of the atoms. So the net magnetization $\mathbf{M} = \mathbf{0}$. When an external magnetic field is applied, the magnetic dipoles tend to align themselves in the direction of the magnetic field. The individual atomic dipole moments point in the same direction. The material becomes magnetized.

Experimentally, one finds that the magnetisation of a paramagnetic material is inversely proportional to the absolute temperature **T**.

$$\chi = C\,\frac{\mu_0}{T} \tag{3.55}$$

This is known as **Curie's law**. The constant **C** is called Curie's constant. Thus, for a paramagnetic material both χ and μ_r depend not only on the material, but also on the sample temperature.

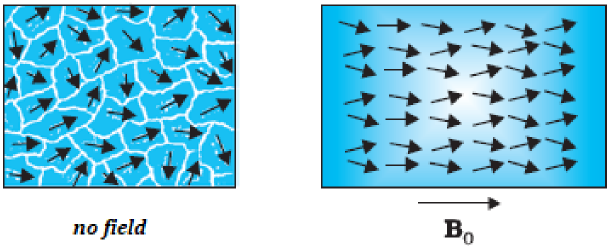


Paramagnetism

3.7.3 Ferromagnetism

Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field. They have strong tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get strongly attracted to a magnet. The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material. However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called *domain*. Each domain has a net magnetisation. Typical domain size is 1mm and the domain contains about 10^{11} atoms. In the first instant, the magnetisation varies randomly from domain to domain and there is no bulk magnetisation.

In a ferromagnetic material the field lines are highly concentrated. In non-uniform magnetic field, the sample tends to move towards the region of high field. We may wonder as to what happens when the external field is removed. In some ferromagnetic materials the magnetisation persists. Such materials are called hard magnetic materials or hard ferromagnets. Alnico, an alloy of iron, aluminium, nickel, cobalt and copper, is one such material. The naturally occurring lodestone is another. Such materials form permanent magnets to be used among other things as a compass needle. On the other hand, there is a class of ferromagnetic materials in which the magnetisation disappears on removal of the external field. Soft iron is one such material. Appropriately enough, such materials are called soft ferromagnetic materials.



Randomly oriented domains, Aligned domains

The ferromagnetic property depends on temperature. At high enough temperature, a ferromagnet becomes a paramagnet. The domain structure disintegrates with temperature. This disappearance of magnetisation with temperature is gradual. It is a phase transition reminding us of the melting of a solid crystal. The temperature of transition from ferromagnetic to paramagnetism is called the **Curie temperature** T_c.

$$\chi = \frac{C}{T - T_c} \quad \text{for } T > T_c \tag{3.56}$$

3.8 Questions

1. What is magnetic field ?
2. Differentiate between magnetic flux density and magnetic intensity
3. State and explain Biot-Savart's law. Use this to find the magnetic induction due to a current carrying circular coil
4. State and explain Biot-Savart's law. Use this to find the axial magnetic induction of a current carrying solenoid
5. What is Lorent'z force equation ?
6. Derive the divergence of magneto-static field
7. Find the curl of magnetic field
8. State and explain Ampere's circuital law
9. What is Meissner effect ?
10. State and explain the terms intensity of magnetisation, magnetic susceptibility, magnetic induction and magnetic permeability
11. What is meant by the solenoidal field ?
12. State and explain Gauss's law in magnetism. Obtain its differential form
13. Obtain the differential form of Ampere's circuital law
14. Distinguish between dia, para and ferromagnetic substances

15. Derive relations connecting **H, M** and **B**
16. Derive the relation connecting relative permeability and magnetic susceptibility
17. What is magnetic vector potential
18. Diamagnetism is universal phenomenon, explain why ?
19. What is magnetic levitation ?
20. A square coil of side l carries a current I. Calculate the magnetic induction at the centre of the coil.
21. A circular coil has a radius of 0.1 m and a number of turns of 50. Calculate the magnetic induction at a point (i) on the axis of the coil and distance 0.2m from the centre; (ii) at the centre of the coil, when a current of 0.1 A flows in it
22. Use Ampere's circuital law to find the expression for magnetic field inside a long solenoid
23. Use Ampere's circuital law to find the expression for magnetic field due to a toroid
24. Write a short note on ferrimagnetism and antiferromagnetism

1. An electron is moving north in a region where the magnetic field is south. The magnetic force exerted on the electron is:
(a) zero (b) up (c) down (d) east (e) west

2. A proton (charge e), traveling perpendicular to a magnetic field, experiences the same force as an alpha particle (charge 2e) which is also traveling perpendicular to the same field. The ratio of their speeds, v_{proton}/v_{alpha}, is:
(a) 0.5 (b) 1 (c) 2 (d) 4 (e) 8

3. A hydrogen atom that has lost its electron is moving east in a region where the magnetic field is directed from south to north. It will be deflected:
(a) up (b) down (c) north (d) south (e) not at all

4. Suitable units for μ_0 are:
(a) tesla (b) newton/ampere (c) weber/meter (d) kilogram·ampere/meter
(e) tesla·meter/ampere

5. Electrons are going around a circle in a counterclockwise direction . At the center of the circle they produce a magnetic field that is:
(a) into the page (b) out of the page (c) to the left (d) to the right
(e) zero

6. Lines of the magnetic field produced by a long straight wire carrying a current are:
(a)in the direction of the current (b) opposite to the direction of the current (c) radially outward from the wire (d) radially inward toward the wire (e) circles that are concentric with the wire

7. Two long straight wires are parallel and carry current in the same direction. The currents are 8.0 and 12A and the wires are separated by 0.40 cm. The magnetic field in tesla at a point midway between the wires is:
(a)0 (b) 4×10^{-4} (c) 8×10^{-4} (d) 12×10^{-4} (e) none of these

8. Solenoid 2 has twice the radius and six times the number of turns per unit length as solenoid 1. The ratio of the magnetic field in the interior of 2 to that in the interior of 1 is:
(a)2 (b) 4 (c) 6 (d) 3 (e) 1

9. A loop of current-carrying wire has a magnetic dipole moment of $5 \times 10^{-4} Am^2$. The moment initially is aligned with a 0.5-T magnetic field. To rotate the loop so its dipole moment is perpendicular to the field and hold it in that orientation, you must do work of:
(a)0 (b) $2.5 \times 10^{-4} J$ (c) $-2.5 \times 10^{-4} J$ (d) $1.0 \times 10^{-4} J$ (e) $-1 \times 10^{-4} J$

10. At a place the vertical and horizontal components of the earth's magnetic field are equal.Then the angle of dip will be equal to :
(a) 30^0 (b) 45^0 (c) 60^0 (d) 75^0 (e) none of these

11. Which of the following is a ferromagnetic ?
(a) aluminium (b) nickel (c) quartz (d) bismuth (e) none of these

12. A small rod of bismuth is suspended freely between the poles of a strong electromagnet.It is found to arrange itself at right angles to the magnetic field.This observation establishes that the bismuth is :
(a) diamagnetic (b) paramagnetic (c) ferromagnetic (d) anti ferro-magnetic (e) none of these

13. A magnetic needle is kept in a non-uniform magnetic field .It experiences :
(a) a force and a torque (b) a force but no torque (c) a torque but no force (d) neither a force nor a torque (e) none of these

14. The force between two magnetic poles is F.If the distance between the poles and pole strengths of each pole are doubled , then the force experienced is:
(a) $2F$ (b) $F/2$ (c) $F/4$ (d) F (e) $4F$

15. A paramagnetic substance of susceptibility 3×10^{-4} is placed in a magnetizing field of $4 \times 10^{-4} Am^{-1}$.Then the intensity of magnetization in the units of Am^{-1} is :
(a) 1.33×10^{8} (b) 0.75×10^{-8} (c) 12×10^{-8} (d) 14×10^{-8} (e) 1.2×10^{-8}

16. Electromagnets are made of soft iron because soft iron has:
(a) low susceptibility and low retentivity (b) low susceptibility and high retentivity (c) high permeability and low retentivity (d) high permeability and high coercivity (e) low permeability and low retentivity

17. Gauss law for magnetism tells us:
(a) that the line integral of a magnetic field around any closed loop must vanish (b) the magnetic field of a current element (c) that magnetic monopoles do not exist (d) charges must be moving to produce magnetic fields (e) the net charge in any given volume

18. A bar magnet is broken in half. Each half is broken in half again, etc. The observation is that each piece has both a north and south pole. This is usually explained by:
(a) Ampere's theory that all magnetic phenomena result from electric currents (b) our inability to divide the magnet into small enough pieces (c) Coulomb's law (d) Lenz' law (e) conservation of charge

19. When a permanent magnet is strongly heated:
(a) nothing happens (b) it becomes an induced magnet (c) it loses its magnetism (d) its magnetism increases (e) its polarity reverses item A circular loop of radius a is formed at a certain point on an infinitely long straight conductor carrying a current of i ampere.Then the magnetic field at the center of the loop is :
(a) $\frac{\mu_0 i}{2\pi a}(\pi - 1)$ (b) $\frac{\mu_0 i}{2\pi\ a}(\pi + 1)$ (c) infinity (d) zero (e) none of these

20. A steel wire of length l has a magnetic moment m. It is then bent into a semicircular arc. The new magnetic moment is :
(a) $\frac{2m}{\pi}$ (b) m (c) $\frac{m}{l}$ (d) $m \times l$ (e) zero

21. A straight wire 2m long carries a current of 2A.What is the force on the wire when it is placed perpendicular to a uniform magnetic field of 0.4×10^{-4}:
(a) $1.6 \times 10^{-4} N$ (b) $1.0 \times 10^{-4} N$ (c) $0.8 \times 10^{-4} N$ (d) zero (e) none of these

22. A thin wire of length 0.2m and a mass $5\mu g$ remains suspended in air between the pieces of a magnet.If the wire is carrying a current of 0.5 A, the strength of the magnetic field is :
a) 50 gauss (b) 5 gauss (c) 0.5 gauss (d) 0.05 gauss (e) 0.005 gauss

23. Two thin long parallel wires , separated by a distance d carry current i A each.The magnitude of the force per unit length exerted by one wire on th other is :
a) $\mu_0 i^2/2\pi d$ (b) $\mu_0 i^2/4\pi d$ (c) $\mu_0 i/2\pi d$ (d) $\mu_0 i/4\pi d$ (e) none of these

24. A short bar magnet with a magnetic moment of 0.5 J/T is placed in a uniform magnetic field of 0.16 T in such a way that the magnetic moment vector is anti-parallel to the direction of the magnetic field.The potential energy of the bar magnet is :
(a) 0.32J (b) 0.16J (c) 0.08J (d) -0.08J (e) -0.32J

25. A bar magnet is released in to a copper ring directly below it.The acceleration of the magnet will be:
(a) equal to the acceleration due to gravity at the place (b) less than the acceleration due to gravity at the place (c) greater than the acceleration due to gravity at the place (d) twice the acceleration due to gravity at the place (e) zero

26. The force on a conductor of length l placed in a magnetic field of magnitude B and carrying a current I is given by (θ is the angle , the conductor makes with the direction of B):
(a) $F = IlBsin\theta$ (b) $F = I^2lB^2sin\theta$ (c) $F = IlBcos\theta$ (d) $F = I^2lsin\theta/B$ (e) $F = I^2lcos\theta/B$

27. A magnetic field CAN NOT:
(a) exert a force on a charged particle (b) change the trajectory of a charged particle (c) change the momentum of a charged particle (d) change the kinetic energy of a charged particle (e) none of these

28. A charged particle is projected into a region of uniform, parallel,E and B fields. The force on the particle is:
(a) zero (b) at some angle $< 90^0$ with the field lines (c) along the field lines (d) perpendicular to the field lines (e) unknown (need to know the sign of the charge)

29. The magnetic field outside a long straight current-carrying wire depends on the distance R from the wire axis according to:
(a) R (b) 1/R (c) $1/R^2$ (d) $1/R^3$ (e) none of these

30. The magnetic field B inside a long ideal solenoid is independent of:
(a) the current (b) the core material (c) the spacing of the windings (d) the cross-sectional area of the solenoid (e) the direction of the current

31. The magnetic force on a charged particle is in the direction of its velocity if:
(a) it is moving in the direction of the field (b) it is moving opposite to the direction of the field (c) it is moving perpendicular to the field (d) it is moving in some other direction (e) never

32. Units of a magnetic field might be:
(a) C·m/s (b) C·s/m (c) C/kg (d) kg/C·s (e) N/C·m

33. You are facing a loop of wire which carries a clockwise current of 3.0A and which surrounds an area of $5.810^{-2}m^2$. The magnetic dipole moment of the loop is:
(a) $3.0Am^2$, away from you (b) $3.0Am^2$, toward you (c) $0.17Am^2$, away from you (d) $0.17Am^2$, toward you (e) $0.17Am^2$, left to right

34. Displacement current was first postulated by:
(a) Maxwell (b) Ampere (c) Hertz (d) Marconi (e) none of these

35. The dimensions of E/B are the same as that of
(a) current (b) charge (c) speed (d) acceleration (e) none of these

36. The velocity of light can be changed by changing its:
(a)frequency (b) wavelength (c) amplitude (d) wave vector (e) none of these

37. Accelerated electrons would produce:
(a) alpha particles (b) beta particles (c) positive rays (d) electromagnetic waves

38. Which of the following types of electromagnetic radiation travels at the greatest speed in vacuum?
(a) Radio waves (b)Visible light (c)Gamma rays (d) All of these travel at the same speed

39. For an electromagnetic wave the direction of the vector $\vec{E} \times \vec{B}$ gives:
(a) the direction of the electric field (b)the direction of the magnetic field (c)the direction of wave propagation (d) the direction of the emf induced by the wave

Note : -

Antiferromagnetic Materials :In an antiferromagnet, unlike a ferromagnet, there is a tendency for the intrinsic magnetic moments of neighboring valence electrons to point in opposite directions. When all atoms are arranged in a substance so that each neighbour is anti-parallel, the substance is antiferromagnetic. Antiferromagnets have a zero net magnetic moment, meaning that no field is produced by them. Manganese oxide (MnO) is one material that displays this behavior. Generally, antiferromagnetic order may exist at sufficiently low temperatures, but vanishes at and above the Neell temperature. Above the Neell temperature, the material is typically paramagnetic, that is, the thermal energy becomes large enough to destroy the microscopic magnetic ordering within the material.

Ferrimagnetic Materials : The macroscopic magnetic characteristics of ferromagnets and ferrimagnets are similar, the distinction lies in the source of the net magnetic moments. A ferrimagnetic material is one that has populations of atoms with opposing magnetic moments, as in antiferromagnetism; however, in ferrimagnetic materials, the opposing moments are unequal and a spontaneous magnetization remains. Ferromagnetic, ferrimagnetic, or antiferromagnetic materials possess permanent magnetization even without external magnetic field and do not have a well defined zero-field susceptibility. Ferrites are usually ferrimagnetic ceramic compounds derived from iron oxides. Magnetite (Fe_3O_4) is a famous example.

Earth's Changing Magnetic Field Earth's magnetic poles do not stay in one place. The magnetic pole in the north today, is in a different place from where it was 20 years ago. In fact, not only does the position of the magnetic poles move, but Earth's magnetic field sometimes reverses direction. For example, 700 thousand years ago, a compass needle that now points north would point south. During the past 20 million years, Earth's magnetic field has reversed direction more than 70 times. The magnetism of ancient rocks contains a record of these magnetic field changes.When some types of molten rock cool, magnetic domains of iron in the rock line up with Earth's magnetic field. After the rock cools, the orientation of these domains is frozen into position. Consequently, these old rocks preserve the orientation of Earth's magnetic field as it was long ago.

(*ii*) Magnetic Field inside a long solenoid

Consider a long straight solenoid having n turns per unit length. Let i be the current flowing in the solenoid. It is experimentally noted that magnetic field outside the solenoid is very small in comparison with the field inside. The lines of induction inside the solenoid are straight and parallel

Consider a closed path *pqrs.* The line integral of magnetic field **B** along path *pqrs* is

$$\oint_{pqrs} \mathbf{B}\cdot d\mathbf{l} = \int_{pq} \mathbf{B}\cdot d\mathbf{l} + \int_{qr} \mathbf{B}\cdot d\mathbf{l} + \int_{rs} \mathbf{B}\cdot d\mathbf{l} + \int_{sp} \mathbf{B}\cdot d\mathbf{l} \quad ...(1)$$

Let $pq = l$. For path pq, **B** and $d\mathbf{l}$ are along same direction.

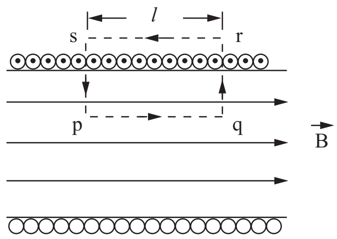


FIG. 10.17

$$\therefore \quad \int_{pq} \mathbf{B}\cdot d\mathbf{l} = \int B\, dl = Bl$$

For paths qr and sp, $\boldsymbol{B}$ and $d\mathbf{l}$ are mutually perpendicular.

$$\therefore \quad \int_{qr} \mathbf{B}\cdot d\mathbf{l} = \int_{sp} \mathbf{B}\cdot d\mathbf{l} = \int B\, dl \cos 90° = 0$$

For path rs, $\mathbf{B} = 0$ (since field is zero outside a solenoid)

$$\therefore \quad \int_{rs} \mathbf{B}\cdot d\mathbf{l} = 0$$

Eq. (1) becomes,
$$\oint_{pqrs} \mathbf{B}\cdot d\mathbf{l} = \int_{pq} \mathbf{B}\cdot d\mathbf{l} = Bl \quad ...(2)$$

By Ampere's law,
$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 \times \text{net current enclosed by path}$$

$$Bl = \mu_0 (nl)\, i$$

$$\therefore \quad B = \mu_0 n\, i \quad ...(3)$$

(*iii*) Magnetic Induction due to a toroid (endless solenoid)

Consider a toroid carrying a current i_0 (Fig. 10.18). Point P is within the toroid while Q is inside and point R outside. By symmetry, direction of **B** at any point is tangential to a circle drawn through that point with same centre as that of toroid. The magnitude of B on any point of such a circle will be constant. Let us consider a point P within the toroid. Let us draw a circle of radius r through it. Applying Ampere's law to this circle, we have

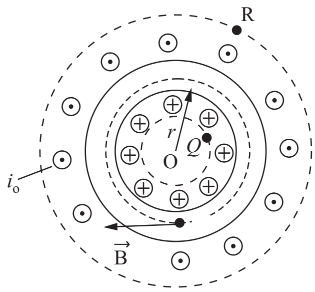


FIG. 10.18

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 i, \quad ... (1)$$

where i is the *net* current enclosed by the circle. Now,

$$\oint \mathbf{B}\cdot d\mathbf{l} = B(2\pi r)$$

and
$$i = N i_0,$$

where N is the total number of turns in the toroid.

$\therefore$ Eq. (l) becomes $B\,(2\pi r) = \mu_0\, N i_0$

or
$$B = \frac{\mu_0}{2\pi}\frac{N i_0}{r}$$

Thus the field B varies with r.

If l be the mean circumference of the toroid, then $l = 2\pi r$, so that

$$B = \frac{\mu_0 N i_0}{l}$$

The field B at an inside point such as Q is zero because there is no current enclosed by the circle through Q.

The field B at an outside point such as R is also zero because net amount of current enclosed in the circle through R will be zero. This is because each turn of the winding passes *twice* through this area enclosed by the circle, carrying equal currents in opposite directions.

The field of a toroid is thus zero at all points except within the core.